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Geometrical errors of coordinate measuring machines

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First published under the title „Unsicherheiten einfach abschätzen“ in QZ Qualität und Zuverlässigkeit, München, 45 (2000) 4, p. 458-464

The uncertainty of measurement contains the estimated geometrical errors of the measuring machine. For some typical measuring tasks the effects of the errors of indication for size measurements on the measurands are used to estimate the task specific geometrical errors. Additionally, the influence of temperature has to be taken into account as a very important further uncertainty component.

Introduction

In every decision concerning the conformance of products based on measurements, their uncertainties have to be considered [1]. The uncertainty characterizes the dispersion of the values that could be attributed to the measurand. There are two fundamental methods of uncertainty assessment [2]. With the first method (A) measurements are repeated several times, with the second method (B) know data like standard resp. expanded uncertainties, maximum permissible errors or distribution functions are used. Method A is more expensive and requires in coordinate measurement repeated measurements of the same workpiece in various positions. Therefore method B is often prefered, if the corresponding data may be gathered with reasonable expense. Determining the relationships between the deviations of the measurands and the error of indication for size measurements for typical measurement tasks, the uncertainty components of the geometrical errors of the CMM may be estimated in a simple manner according to the method B.
The described method is one possible alternative to other ones, which are currently under discussion like the “virtual coordinate measuring machine” [3] or “uncertainty assessment by simulation” [4].

Error of indication for size measurement

The most well known geometrical deviation of coordinate masuring machines is the error of indication for size measurement. Their maximum permissible errors are used to compare the accuracy of various machines. In acceptance tests the maximum permissible errors stated by the manufacturer of the CMM are not allowed to be exceeded, if the stated environmental conditions are retained. In reverification tests, the user may state his own maximum permissible errors that satisfy his requirements [5-7].
The errors of indication for size measurements are usually verified measuring gauge blocks, alternatively step gauges, ball bars and ball plates. The tests of singular components of the geometrical errors with laser interferometers resp. material standards are expensive and remain usually for the manufacturers internal tests. These tests are not typically carried out by the user.
Ball plate measurements may not only be used to test the errors of indication for size measurement, but also - using the distances of the ball centers - to calculate e.g. the errors of straightness and squareness [5, 6]. Therefore the maximum permissible error of indication for size measurement is limiting also these errors and may be used to derive approximated values for the errors of form, orientation and position.
The maximum permissible error of indication for size measurements is usually stated in the form:

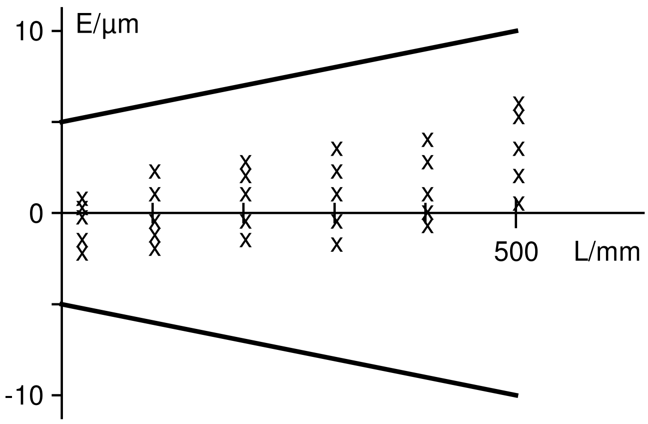
$$MPE_E \leq \left(A + \frac{L}{K} \right) \mu m \qquad \text{(measured size } L \text{ in mm)} \qquad (1)$$

with A as a constant, and L/K as an length dependent component [7]. The constant factor represents the probing uncertainty of the surface of the material standard. The length dependent component expresses the geometrical errors of the CMM.
The errors of indication for size measurement typically lie in the center between the two straight lines (figure 1), representing the maximum permissible errors. They occupy about the half range. In no case

they are distributed evenly across the entire range. Therefore a normal distribution of the errors may be assumed.

Figure 1:
Example for errors of indication of a CMM for size measurements MPE_E with the maximum permissible error

$$MPE_E \leq \left(5 + \frac{L}{100}\right) \mu m$$



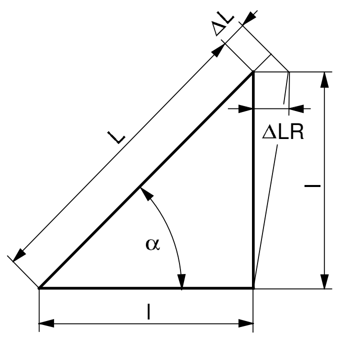
Measuring the distance between two parallel planes, the length dependent component L/K immediately represents the uncertainty contribution of the geometrical error of the CMM. Its maximum permissible error is derived dividing the nominal value of the length L by the factor K. The nominal value L is valid for all measurands that may be interpreted as a length or distance.
By contrast, the constant A may not immediately be used as a uncertainty contribution, because it is valid only for the two-point-measurement between even surfaces. Depending on the effect of local errors of form and the calculation of the mean value for several probing points, the uncertainty contribution of other measurings tasks may be larger or smaller respectively. For the distance of the centre points of two circles it is usually substantially smaller if the circles were measured with noticeably more than the mathematically required minimal number of points. The probing uncertainty in this case may be assessed by a series of repeated measurements, where every time other points of the surface are touched. Alternatively the uncertainty may be assessed from one measurement itself [8, 9].

Errors of orientation and form

The geometrical errors of the CMM affect not only measurements of size, but also other measurements in different ways. An error of squareness e.g. may be determined approximately with a length measurement in a diagonal direction at an angle of 45° (figure 2). In the least favorable case the error of indicaton for size measurement ΔL is all alone caused by the error of squareness ΔL_R. Because of $L\cos\alpha = l$ and $\cos^2\alpha = 0{,}5$ for $\alpha = 45°$ it is twice as great as the error of indication for size measurement of the length l of the shorter side of the angle:

$$\Delta L_R \leq \frac{\Delta L}{\cos\alpha} = \frac{2l}{K} \quad (2)$$

Figure 2:
Error of indication for size measurement ΔL and error of squareness ΔL_R



In the case of parallel straigth lines their inclination is measured in a common plane (figure 3). The same applies to rotational errors occuring in the direction of movement. The error may be determined approximately with a length measurement in a diagonal direction at an angle of 45°. In the least favorable case the error of indication for size measurement is all alone caused by the error of parallelism ΔL_P or rotation respectively. On account of the same geometrical relationships between L, l and $\cos^2\alpha$ at α=45° the error ΔL_P is twice as great as the error of indication for size measurement of the length l of the side of the angle:

$$\Delta L_P \leq \frac{\Delta L}{\cos\alpha} = \frac{2l}{K} \qquad (3)$$

For l is to apply the smaller value of the distance of the two geometrical features or the length of the tolerated feature, because the error in the case of very small dimensions must approach zero.

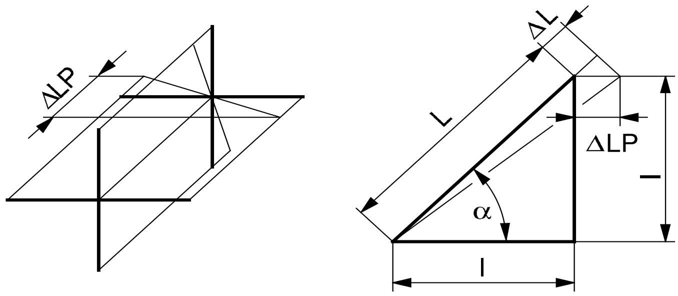


Figure 3:
Error of parallelism (inclination) resp. error of rotation; left side spatial situation, right side error of indication for size measurement ΔL and error of orientation ΔL_P

The error of straigthness may be determined approximately with a length measurement in a diagonal direction at an angle of 45°. In the least favorable case the error of indication for size measurement is all alone caused by the error of straigthness ΔF_G. Because of $L\cos\alpha = l/2$ and $\cos^2\alpha$=0,5 for α=45° it is just as great as the error of indication for size measurement of the length l of the straight line:

$$\Delta F_G \leq \frac{\Delta L}{\cos\alpha} = \frac{l}{K} \qquad (4)$$

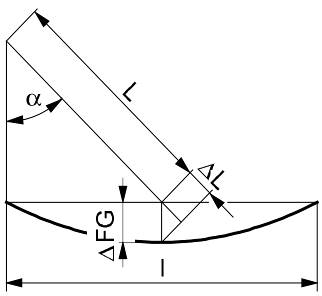


Figure 4:
Error if indication for size measurement ΔL and error of straigthness ΔF_G

Other geometrical properties

The above described procedure may be also applied for further geometrical properties to derive relationships to the error of indication for size measurement. Table 1 shows such approximation formulas to assess the uncertainty contributions of the geometrical errors of CMM. In each case, it’s a matter of maximum permissible errors limiting normally distributed deviations. The standard uncertainties are calculated dividing these values by the factor k=2 [2].
At the properties parallelism, squareness and inclination the datum is required to be longer than the tolerated feature. In the reverse case the maximum permissible error increases with the relation of these length.

Table 1: Maximum permissible geometrical errors of a CMM for various geometrical properties; with the factor K of the maximum permissible error of indication for size measurement $MPE_E=(A+L/K)$ µm (L in mm); the deviation of the angel $\Delta\hat{\alpha}$ in radiant may be converted through multiplication with 180°/π into degrees, trough further multiplication with the factor 60 into minutes etc.

Geometrical property	Maximum permissible error (in µm)	Comment
Length, distance, diameter and position (within a plane)	$\Delta L \le \frac{L}{K}$	*L* Nominal value of the property resp. theoretical value of the position tolerance
Position (within the space)	$\Delta L \le \frac{1}{K}\sqrt{L^2 + l^2}$	*L* Theoretical value of the position tolerance *l* Greatest nominal value of the geometric feature perpendicular to the theoretical value (diagonal)
Concentricity and symmetry between points	$\Delta L_K \le \frac{D}{2K}$	*D* Greatest nominal value of the diameter resp. width
Coaxiality and symmetry (within a plane)	$\Delta L_K \le \frac{1}{K}\sqrt{\frac{D^2}{4} + L^2}$	*D* Greatest nominal value for the diameter resp. width *L* Nominal value of the smaller length
Parallelism (inclination and crosswise inclination) and rotation	$\Delta L_P \le \frac{2L}{K}$	*L* Smaller nominal value for the length of the measured feature resp. the perpen-dicular distance of the both features (the longer feature is the datum)
Squareness	$\Delta L_R \le \frac{2L}{K}$	*L* Nominal value of the length of the shorter side of the angel (the longer side of the angel is the datum)
Inclination	$\Delta L_N \le \frac{2L}{K}\sin\alpha$	*L* Nominal value of the side of the triangel opposite to the angel (the longer side of the angel is the datum) α Nominal value of the angel
Deviation of angel (radiant)	$\Delta\hat{\alpha} \le \frac{2}{10^3 K}\sin^2\alpha$	α Nominal value of the angel
Straightness	$\Delta F_G \le \frac{L}{K}$	*l* Nominal value of the length of the straight line
Flatness	$\Delta F_E \le \frac{1}{K}\sqrt{5l^2 + L^2}$	*l, L* Nominal values of the shorter and the longer side of the plane
Roundness	$\Delta F_R \le \frac{D}{K}\sqrt{\frac{26}{4}}$	*D* Nominal value of the diameter
Zylindricity	$\Delta F_Z \le \frac{1}{K}\sqrt{\frac{26}{4}D^2 + 10L^2}$	*D* Nominal value of the diameter *L* Nominal value of the length

As shown in the figures 2 to 4, all geometrical errors of the CMM may affect the error of indication for size measurement. Using singular length measurements, it is not entirely clear if the error of indication is caused e.g. by scale, rotation, squareness or straightness errors. However, because all error components are taken into account with their full values, the standard uncertainties are expected more likely too large than too small.
Due to the normally distributed errors of indication for size measurement, the standard uncertainty is in every case half as large as the maximum permissible errors according to table 1. Its application of course presupposes the regular verification of the CMM and the operating conditions to be always held.

The formulas in table 1 cover the major part of practical measuring tasks. For other geometric features like gears, threads or non-regular surfaces the following rules may be stated:

1. An error of size is not greater than the length dependent component *L*/*K*.
2. An error of orientation or position is maximally twice as large as the length dependent component *L*/*K*, if the datum is the longer resp. larger feature.
3. An error of form is not greater than four times the length dependent component *L*/*K*.

For *L* the largest spatial diagonal of the geometric feature resp. of the workpiece may be used.

Effect of temperature

When estimating the temperature influence on the measurand generally a homogenous temperature within the workpiece is presupposed. Then the temperatur has no effect on the errors of form and orientation, but only on absolute quantities like sizes and distances, with the position tolerance belonging to it, too.
Independently of the geometry of the workpiece and the geometrical property, the errors caused by temperatur deviations always may be assessed using certain rules [10], if for the length *L* is set the longest expansion of the workpiece (spatial diagonal). If the geometrical property is referred to a geometric feature much smaller than the entire workpiece, then its spatial diagonal may be used.
If the temperature within the workpiece is not homogenous, the temperature difference Δt should be determined. It causes a difference of length $\Delta L_{\Delta t}$, that depends on the nominal value of length L and the expansion coefficient α_w. The most unfavorable case is estimated using the length L of the spatial diagonal of the workpiece resp. the geometric feature:

$$\Delta L_{\Delta t} = L\, \alpha_w\, \Delta t \tag{6}$$

This deviation may be taken into account as an additional uncertainty component in measurements of form, orientation and position. Its effects on the geometric properties are the same as described above for ΔL. In this way also the influence of different temperatures within the workpiece may be estimated as an uncertainty component.

Example: distance of holes

At an workpiece the distance of two holes is to be measured (figure 5). The mathematical model may be formulated in the simplest way as the coordinate difference between x_1 and x_2:

$$L = x_2 - x_1 \tag{6}$$

Additionally there are to be taken into account the geometrical error ΔL of the CMM and the temperature deviation ΔL_T. The complete equation then is stated as:

$$L = x_2 - x_1 - \Delta L - \Delta L_T \tag{7}$$

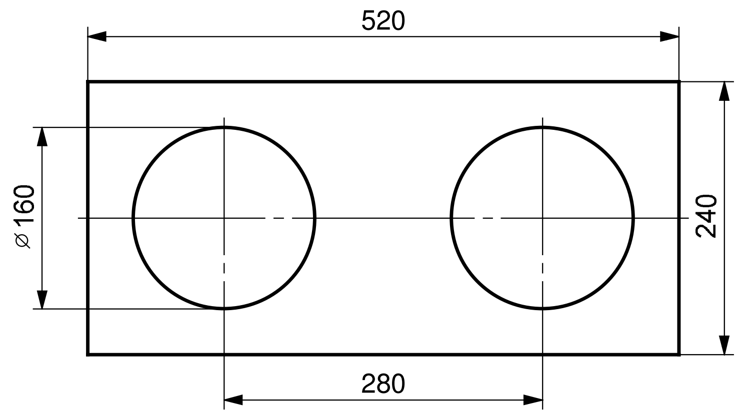


Figure 5:
Measurement of the distance of the centres of the holes

The combined standard uncertainty derives from the standard uncertainties $u(x_i)$ of the influence quantities by the application of the propagation rule on equation (7):

$$u(L) = \sqrt{u^2(x_1) + u^2(x_2) + u^2(\Delta L) + u^2(\Delta L_T)} \tag{8}$$

Presupposing a homogenous distribution of the positions of the probing poins on the entire circumference, there exists a simple relationship between the number of point and the expanded uncertainty U(x) of the centre point coordinates [8]. The uncertainty may be estimated by multiplying the standard deviation s of the mean circle with the corresponding factor from table 2. The standard uncertainty $u(x)$ is as half as large, if the deviations are normally distributed.
The standard deviation s contains the effects of the errors of geometry and of probing of the CMM within the area of the holes, of the number of probing points and that of the local deviations of form of the probe and of the surfaces. Especially the latter is very anvantagous, compared to other procedures [3, 4] that do not consider the local deviations of form at all.

Table 2: Expanded uncertainties of the centre $U(x)$ and the diameter $U(D)$ of a mean circle as a functin of the number of points [8] with the following conditions:
- Points with equal distances at the whole circumference
- Random deviations of the probing points from the mean circle
- Level of confidence P=95%
- Standardized on the standard deviation s=1

Number of points n	Centre $U(x)/s$	Diameter $U(D)/s$
4	8,98	12,71
6	1,84	2,60
8	1,29	1,82
12	0,92	1,31
20	0,67	0,94
50	0,40	0,57
100	0,28	0,40
200	0,20	0,28
500	0,12	0,18
1000	0,09	0,12

The standard uncertainty $u(\Delta L_T)$ of the temperature effects may be estimated according to [10]. The expanded uncertainty of the distance of the holes for the level of confidence of 95% is calculated by multiplying the combined standard uncertainty $u(L)$ with the coverage factor k=2 [2].
Table 3 shows the uncertainty budget completely relying on the method B of the Guide, without any repeated measurements according to the method A. In the equation the temperature component ΔL_T is already substituted by the influence quantities of the workpiece and the scale (temperatures and expansion coefficients) [10].
Both circles were measured with n=50 points and standard deviations of s=5µm. According to table 2 there are expanded uncertainties of $U(x1)$= $U(x2)$=2,0 µm. The CMM has the maximum permissible error if indication for size measurement given in figure 1. With L=280 µm the legth dependent component of the maximum permissible error of the distance is ΔL=2,8 µm.
In the case of a number of probing points noticeably higher than the mathematically required minimal number, the deviations from the mean circle may contain systematic components, especially if the surface is characterized by local deviations of form. Then the standard deviation of the random components may be estimated, which yields to a smaller uncertainty of the centre points [8, 9].
Measuring both holes with the same probe, it has no influence on the distance. But, using different probes the influence of their centre points on the distance has to be taken into account. The uncertainties may be estimated with the standard deviation at probe system qualification [8].

Measuring the diameters of the holes, the influence of the probe diameter must be considered in every case. The effect of handling and clamping the workpiece may be figured out by repeated measurements, but it should be negligibly small.

Table 3: Budget of uncertainty of measurement for the distance of the hole centres from figure 5 without probe effect; expansion factor k=2, level of confidence P=95%

Function: $L = (x_2-x_1)*[1-\alpha*(t_W-20°C)+\alpha_S*(t_S-20°C)] + \Delta L$

L	Distance of the hole centres
x_1	Coordinate of the centre of hole 1
x_2	Coordinate of the centre of hole 2
α_W	Expansion coeffizient of the workpiece (steel)
t_W	Temperature of the workpiece
α_S	Expansion coeffizient of the scale (float glass)
t_S	Temperature of the scale
ΔL	Geometrical error of the CMM

Quantity X_i	Unit $[X_i]$	Value x_i	Distribution	Permiss. error Δx_i	Standard uncertainty $u(x_i)$	Sensit.- coeffiz. c_i	Uncertainty contribution $u_i(L)$
x_1	mm	97.0013	Normal	0.0020	0.0010	-1.0000	0.0010
x_2	mm	377.0042	Normal	0.0020	0.0010	1.0000	0.0010
α_W	µm/m/K	12.0	Rectang.	2.4	1.4	-0.0003	0.0004
t_W	°C	21.0	Rectang.	1.0	0.6	-0.0034	0.0020
α_S	µm/m/K	7.8	Rectang.	0.5	0.3	0.0003	0.0001
t_S	°C	21.0	Rectang.	1.0	0.6	0.0022	0.0013
ΔL	µm	0.0	Normal	2.8	1.4	0.0010	0.0014
L	mm	280.0017					0.0031

Result of measurement and expanded uncertainty: L = 280.0017 ± 0.0063

Conclusions

The geometrical errors of coordinate measuring machines affect the errors of indication for size measurements in various ways. For a lot of geometrical properties these relationships may be used to estimate their maximum permissible errors from the maximum permissible error of indication for size measurements. The uncertainty of coordinate measurements in this way may be completely assessed with minimal expense.
The simplicity of the method implies a possible over-estimation of the uncertainty, which may be tolerated for the most cases. The uncertainty may be assessed to small, if the mathematical model of the measurement is incomplete or wrong. Here a careful analysis of the problem is essential.

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